Advances in Complex Systems

DENSE GRANULAR MEDIA AS ATHERMAL GLASSES.

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I briefly describe how mean-field glass models can be extended to the case where the bath and friction are non-thermal. Solving their dynamics one discovers a temperature with a thermodynamic meaning associated with the slow rearrangements, even though there is no thermodynamic temperature at level of fast dynamics. This temperature can be shown to match the one defined on the basis of a flat measure over blocked (jammed) configurations. Numerical checks on realistic systems suggest that these features may be valid in general.

1. Glasses and Dense Granular Matter

An ensemble of many elastic particles of irregular shapes at low temperatures and high densities forms a glass — that is, an out of equilibrium system having a relaxation timescale that grows as the system ages. Granular matter would be just an example of this, albeit a rather special one, in that the thermal kinetic energy $\sim k_B T$ per particle is negligible and that the gravity field plays an unusually important role. What in fact distinguishes granular matter from a glass at zero temperature and very high pressure is the non-thermal manner in which energy is supplied to the grains (vibration, tapping or shearing) and lost by them (inelastic collisions). It is because of this difference that we refer to the granular-matter/glass *analogy*, rather than *identity*.

This analogy was already described at the experimental level by Struik [1], who presented settling powders as aging systems on an equal footing with glasses, and made more explicit by the Chicago group [2,3]. From the theoretical point of view, there has been a free exchange of ideas and models from one field to the other. (See Refs. [4,6,7,8,9,10,11,12,13,14,15] for some examples.)

We can thus view the conceptual passage from glasses to dense granular matter as divided in two steps. The first consists of studying glass models in contact with a heat bath of very low temperature, under a strong gravity field, and considering them from the point of view of the quantities that are measured in granular matter experiments. The second step consists of focusing on which new features are brought in by the non-thermal agitation and friction mechanisms.

As far as the compaction dynamics is concerned, the second question is usually considered less relevant: Thus, in the models, vibration is often substituded by a thermal agitation satisfying detailed balance; for example in lattice models by letting particles move upwards with probability p_{up} and downwards with probability $1 - p_{up}$ (a thermal bath with temperature $\propto \ln^{-1}[\frac{p_{up}}{1-p_{up}}]$). However, if the recent analytical developments in glass theory [16] are to be applied to granular matter, it is unavoidable to face the question of the non-thermal nature of the energy exchange mechanism, as we shall see below.

2. Cage and structural temperatures in glasses.

A dozen years ago, a family of models was identified as being schematic *mean-field* versions of structural glasses, somewhat like the Curie-Weiss model is for ferromagnets. Above a critical temperature, the dynamics of these models is given by mode-coupling equations, or generalisations of them. These equations predict that the relaxation of all quantities proceeds in two steps: a rapid one given the movement of particles in a 'cage' formed by its neighbours, and a slow one generated by the rearrangement of cages: the structural or α-relaxations. As the temperature is lowered, the structural relaxation time becomes larger and larger, and it diverges at the critical temperature. (This transition is in fact smeared in real life, a fact that can be understood within the same framework).

If the systems are quenched below the transition temperature, they fall out of equilibrium: the structural relaxation time is not constant but grows with the ('waiting') time elapsed after the quench, a phenomenon known as aging. Alternatively, one can submit a system below the critical temperature to forces that, like shear stress, can do work continuously. The surprising result in this case is that aging is interrupted (see Refs. [11,17]): the structural relaxation time saturates to a driving-force dependent value. This rejuvenation effect is known as *shear-thinning* or *thixotropy*, depending on whether it applies in the liquid or the glass phase.

Below the transition temperature, the system is out of equilibrium, either because it is still aging or because of the external forces in the driven case. An old idea [18] in glass physics is to consider that the structural degrees of freedom remain at a higher temperature (of the order of the glass temperature), while the cage motion thermalises with the bath. In order to make this idea sharp, we can ask what would be the reading of a thermometer coupled to the glass. One can show [19] that this is related to the ratio of fluctuations and dissipation, as we now describe.

Consider an observable A, with zero mean and with fluctuations characterised by their autocorrelation function $C_A(t_w+t, t_w) =< A(t+t_w)A(t_w) >$. Let us denote $\chi_A(t_w + t, t_w)$ the response $\delta < A(t_w + t) > /\delta h$ to a field h conjugate to A, acting from t_w to $t_w + t$.

If above the glass temperature we plot χ_A versus C_A using t as a parameter, we obtain a straight line with gradient $-1/T$: the fluctuation-dissipation theorem. For a system aging or subjected to nonconservative forces below the glass temperature

we can still make the same plot, using t (and t_w, in the aging case) as parameters. It turns out that one obtains a line with *two* straight tracts: for values of C_A, χ_A corresponding to fast relaxations the gradient is $-1/T$, while for values corresponding to the structural relaxation the gradient is a constant $-1/T_{dyn}$. The effective temperature T_{dyn} so defined is in fact the temperature read in a thermometer coupled to A tuned to respond to the slow fluctuations [19]. Most importantly, it is observable-independent within each timescale.

These facts were originally found in the mean-field/mode-coupling approximation for glassy dynamics, and later verified numerically (at least within the times, sizes and precision involved) for a host of realistic glass models [16,17,20].

The appearence of a temperature T_{dyn} for the slow degrees of freedom, immediately suggested a comparison with an idea proposed by Edwards originally for granular matter [4,5]. For a glass at very low temperatures it can be stated as follows: as the glass ages and its energy $E(t)$ slowly decreases, the value of all macroscopic observables at time t can be computed from an ensemble consisting of all blocked configurations (the local energy minima) having energy $E(t)$, taken with *equal statistic weights*. This ensemble immediately leads to the definition of an entropy $S_{Edw}(E)$ as the logarithm of the number of blocked configurations, and a temperature $T_{Edw}^{-1} = dS_{Edw}/dE$ [21].

Now, for the mean-field/mode coupling models, *it turns out that T_{dyn} and T_{Edw} coincide*, and, furthermore, Edwards' ensemble defined above yields the correct values for the observables out of equilibrium [11]. This has been recently checked for more realistic (nonmean-field) models [22,13].

3. Structural temperature in (dissipative) granular matter.

In order to see what new features are to be found in granular matter, we start with the mean-field/mode coupling models, modifying them in two ways: Firstly we allow for frictional forces that are non-linear, complicated functions of the velocities. Secondly, we drive the systems with forces that do not derive from a potential ('shear-like'), or are strong and periodic in time (vibration and tapping). We expect that the mean-field glass models thus modified will be minimal mean-field granular matter models.

We measure as before correlations and responses, and, in particular diffusion $< |x(t+t_w) - x(t_w)|^2 >$ and mobility $\delta |x(t+t_w)|/\delta f$, where f is a force acting from t_w to $t + t_w$. The vibrated or tapped case has to be measured 'stroboscopically': in order to avoid seeing oscillations we only consider times that correspond to integer numbers n, n' of cycles

$$C(t_n, t_{n'}) =< x(t_n) x(t_{n'}) > \tag{3.1}$$

$$\chi(t_n, t'_n) = \frac{\delta < x(t_n) >}{\delta f} \tag{3.2}$$

where the force acts during an integer number of cycles from $t_{n'}$ to t_n.

In the thermal case we found that above the glass temperature the comparison of correlations and responses yields the bath's temperature (as it should, in an equilibrium situation), and below the glass phase in addition a temperature T_{dyn} for the slow degrees of freedom. For the granular athermal case, this already poses a problem, as not even in a liquid-like fluidised state do we have a well defined temperature! (In other words, a parametric χ_A versus C_A plot will not give a straight line independent of the observable A). This will also be true for the 'cage' motion [23] in the dense regime.

Surprisingly enough, the next step came from the treatment of quantum glasses at zero temperature at the mean-field level. It turns out [24,25] that these systems obey a quantum fluctuation-dissipation theorem in the cage motion, but a classical one in the slow, structural motion: the nature of the bath is irrelevant (in the sense of renormalisation group) as far as the slow motion is concerned. In the context of granular matter, a similar reasoning [26,27] shows that while there is no well-defined dynamic temperature associated to the fast relaxations — and in the fluidised regime these are the only relaxations present, the slow structural relaxations still satisfy a fluctuation-dissipation relation, with an observable-independent temperature T_{dyn} (see figs 1,2).

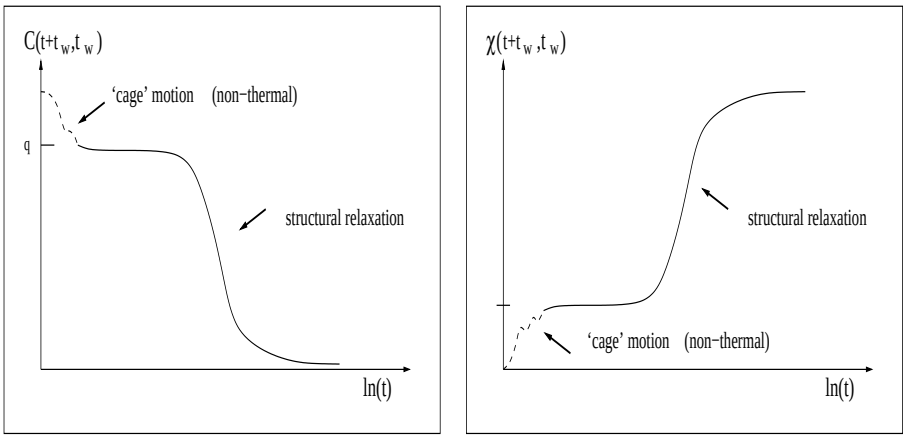

Fig. 1. Sketch of a correlation (left) and a response (right) vs. time.

Once these questions have been clarified at the level of mean-field/mode-coupling models, one feels encouraged to check them numerically and experimentally in realistic systems [28]. Recently [29], a simulation of granular matter subjected to shear has given evidence for the existence of a structural temperature. This dynamical temperature is calculated from the relation between diffusivity and mobility of different tracers, and its independency of the tracer shape is checked. The interest of this setting is that it can be implemented experimentally.

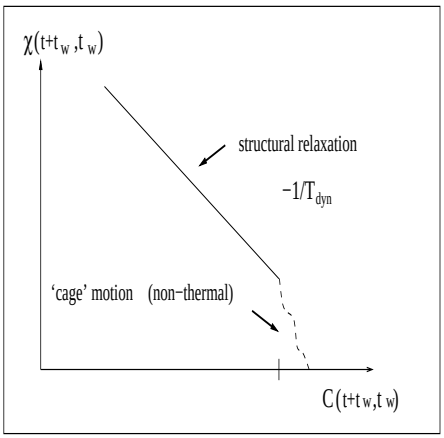

Fig. 2. Effective temperature plot. The dashed tract (fast relaxations) is not straight and is observable dependent. The full line (structural relaxations) is straight and defines an observable-independent temperature.

Within the same model, a direct computation a thermodynamic temperature defined on the basis of the blocked configurations has yielded very good agreement with the dynamical temperature.

4. Conclusion

In conclusion, there has been progress in the theory of statistical ensembles for dense granular matter.

- We have a better idea of how we should understand them, and of their possible domain of validity.
- We have solvable models, and a limit in which we can check if and when these ideas hold strictly.
- We have suggestions for experiments that will test the validity of the approach in each case.

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[21] A brief historical note:
Edwards himself never defined such a temperature, and indeed one of the main aims of his articles cited above is to advocate the use of volume rather than energy as the relevant variable - leading to 'compactivity' (a kind of pressure) rather than to temperature.

However, in those articles he takes for granted a flat average over blocked configurations: this is the aspect of those works which interests us here. Once an 'ergodic' notion is defined, one can choose which ensemble is the most appropiate in each case (fixing volume and/or energy, etc), and the thermodynamic variables follow.

'Edwards' temperature' here is short for: 'the temperature that is obtained from a flat ensemble à la Edwards'.
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